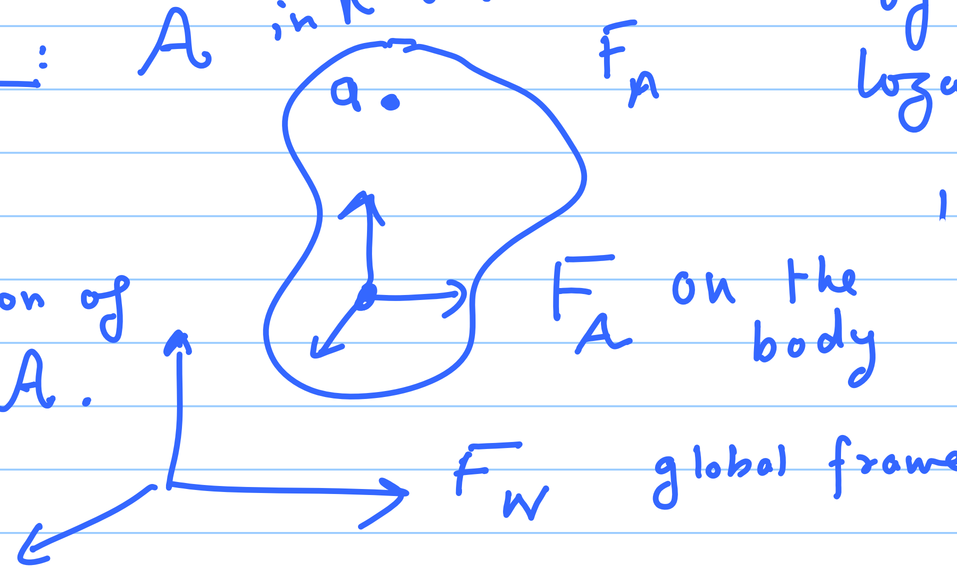
Note Title 9/21/2006

Lecture 6

1) C-space
2) metrics over C-space
3) Basic path planning problem ← was the motivating problem for the notion of C-space Lozano Perez @ MIT 1978

RIGID Body: A in R^2 or R^3



Configuration: specification of each point on A.

min. # of parameters

rep. || Config: spec. of F_A w.r.t F_W

This rep. depends on F_A.

$a \in A$

$a(q) \in R^3$

q : config. $\quad A(q) \subset R^2$ or R^3

$T = \left[\begin{array}{c|c} R & P \\ \hline 0 & 1 \end{array}\right]$

$\in R^N \times SO(N) = SE(N)$

$N = 3 \rightarrow SO(3) \rightarrow$ manifold 3 dim

for rigid bodies

C-space = space of all q : $SE(N)$

$N = 3 \quad SE(N)$ is a six dim manifold

free flying robot

locally $R^3 \times R^3$ (α, β, γ)

$\boxed{g_i(q) = 0}$ for articulated bodies: open chain

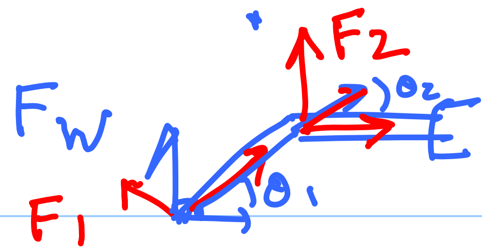
$i = 1, \ldots n$

$S^1 \times S^1 \times S^1 \neq SO(3)$

holonomic constraints

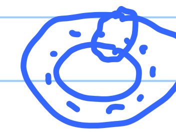
$g_i(q, \dot{q}) = 0$

non-holonomic



$SO(2) \times SO(2)$

C-space $= S^1 \times S^1 =$ Torus(2)

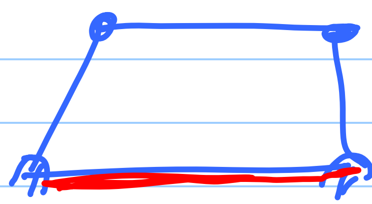


apply all the material re: manifolds

<u>Constraint</u>

|| for closed chains, more complicated

a few works exist for closed chains to char. c-space.



exercise: 4-bar linkage: what is its c-space? ~~PB~~

non-trivial

<u>Grübler's formula</u>:

$\underline{\underline{M = 1}}$

M = # of degrees of freedom of the mechanism
= "dim of C-space"

k links → rigid bodies → in $\mathbb{R}^2$ or $\mathbb{R}^3$ $\quad k = 4$

$k-1$ movable, 1 link fixed

N = # of dofs of each link as an independent body $= 3$

f_i = # of dofs at each joint $= 1$

$$M = N(k-1) - \sum_{i=1}^{k} (N - f_i)$$

metrics over c-space :

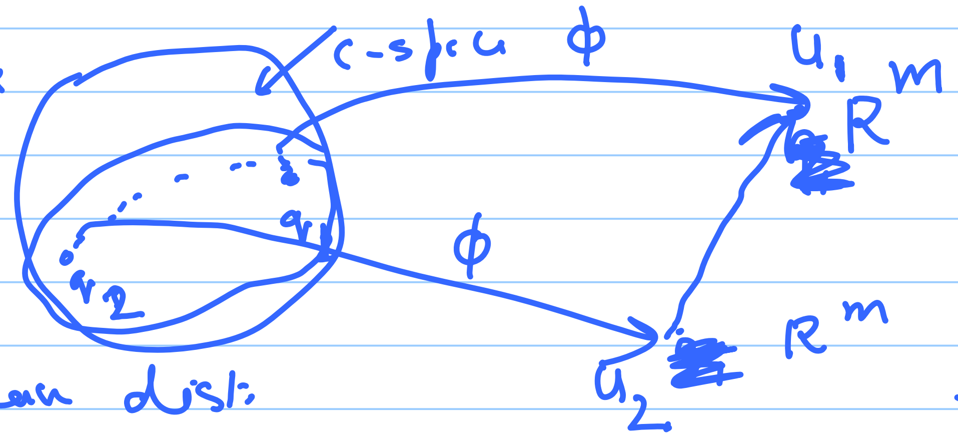
how far apart are two configs ??

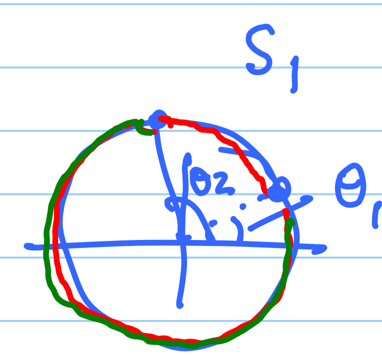
"Computation time"

$$d(q_1, q_2) = ??$$

① c-space is a m dim. mainfold

we could use the co-ord. under ϕ and take the Euclidean dist.

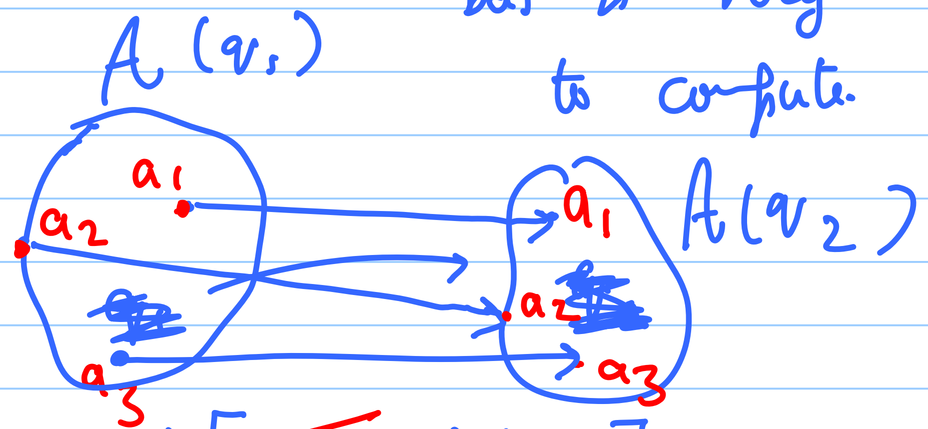




$$d(q_1, q_2) = \| u_1 - u_2 \|$$

"Not rep. of actual "motion" but is very efficient to compute.

2)



$$d(q_1, q_2) = \cancel{d\left[A(q_1), A(q_2)\right]}$$

$$\max_{a \in A} \| a(q_1) - a(q_2) \|$$

may be quite diff to compute

for arb. cons Careful: some of these may not satisfy all prop. of a metric

"pseudo-metric"

③ Hausdorff distance

$\frac{d^2\gamma}{ds^2}$

exercise: check if they satisfy these properties?

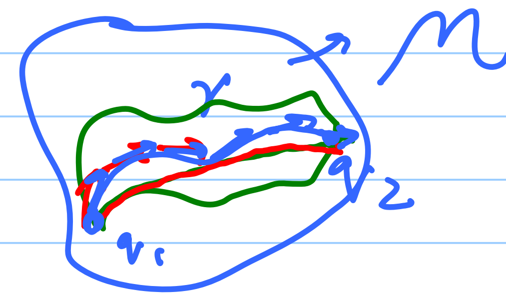
$\gamma(0) = q_1$

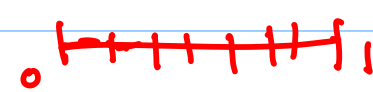
$\gamma(1) = q_2$ $\gamma : [0,1] \to M$

3) Reimannian metric:

$$L(\gamma) = \int_0^1 \left\| \frac{d\gamma}{ds} \right\| ds$$

norm over target space





e.g. config. ~~space~~ difficult in general

$$d(q_1, q_2) = \inf_\gamma L(\gamma)$$

metric sugg. by Mason:

rigid body:

Trans + Rotation

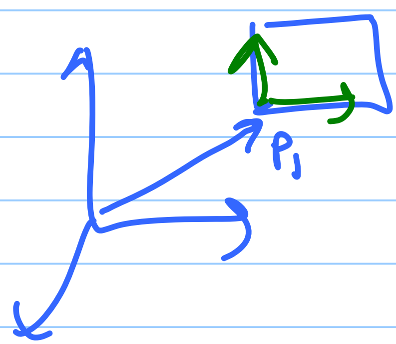
$\downarrow q_1 \qquad\qquad \downarrow q_2$

$P_1, R_1 \qquad\qquad P_2, R_2 \xrightarrow{\phi} \begin{pmatrix}\alpha_2\\ \beta_2\\ \gamma_2\end{pmatrix}$

$R_1 \xrightarrow{\phi} \begin{pmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{pmatrix}$

$\neq$

F_A, F_W are fixed



$$d = \underline{\underline{\| P_1 - P_2 \|}} + \underline{\lambda} \left\| \begin{pmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{pmatrix} - \begin{pmatrix}\alpha_2\\ \beta_2\\ \gamma_2\end{pmatrix} \right\|$$

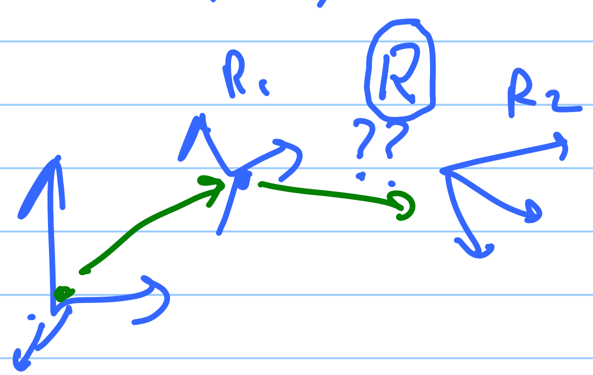
units

|| dep. on parametrization of c-space

|| SE(N) : There is no bi-~~frame-invariant~~ metric. scale invariant.

I will dig up the precise reference. [Park ...]

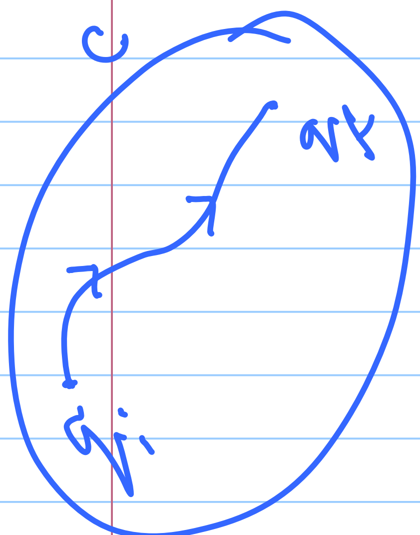
$$d(R_1, R_2) = \left\| \begin{pmatrix} n_x \theta \\ n_y \theta \\ n_z \theta \end{pmatrix}_{w} \right\| \qquad R_{\hat{k}}(\theta)$$

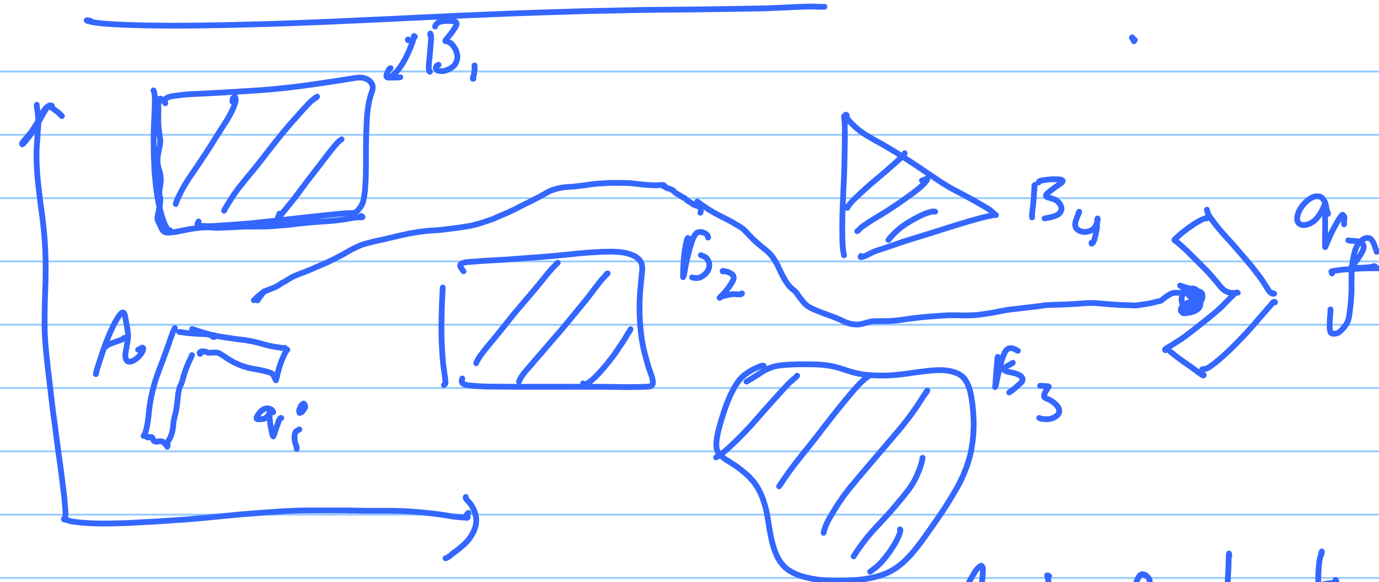


$$R R_1 = R_2 \implies \boxed{{}^{w}R = R_2 R_1^{-1}}$$

$$R_1 \, {}^{R_1}R = R_2 \qquad {}^{R_1}R = R_1^{-1} R_2$$

Basic path plan:





Given: B_i's, A, initial config q_i of A, final config q_f of A, find a collision free path that connect q_1 to q_2.

A is compact

B_i are closed (unbounded obstacles allowed in general)

1966 → oxford

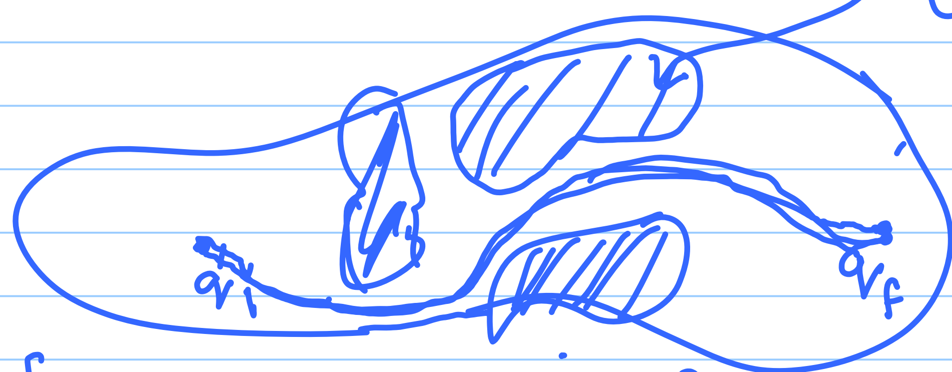
1968 → space : robot arm
Caltech (Udupa)

$\boxed{\text{formal prob. state : Lozano Perez ✓}}$

Basic path planning

✓ 1) Transform the problem to C-space of A

Corr. to those q that result in coll with B_i



$$CB = \bigcup_{i=1}^{4} CB_i$$

$$CB_i = \{ q : A(q) \cap B_i \neq \phi \}$$

2) Map $B_i \longrightarrow CB_i$ are C-obstacles

$C_{free} = C - \{\cup CB_i\}$

Next lecture: how to compute

2nd cont. Ch3 Latombe alg. checkset

3) determine a curve $\tau : [0,1] \longrightarrow C_{free}$

τ is continuous mapping

path

Basic path plng prob.

key issue: 1) C-space in high dim. for most robots of pract. int. (arm)

Branching

2) CB_i are highly complex, non-linear mappings, difficult to compute explicitly in an efficient way.

recall

Ruler folding NP complete. input is "no of links" → exponential in dim of c-space terminology: $C_{obs} = \bigcup CB_i$

$C_{free} = C - C_{obs}$

Q: topological structure/nature of CB_i ?? given B_i

2) B is compact : is CB_i ?? Yes

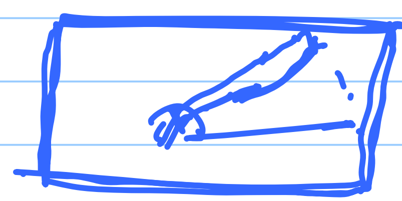
$$CB_i = \{ q : A(q) \cap B_i \neq \phi \}$$

1) B is closed : is CB_i ?? Yes

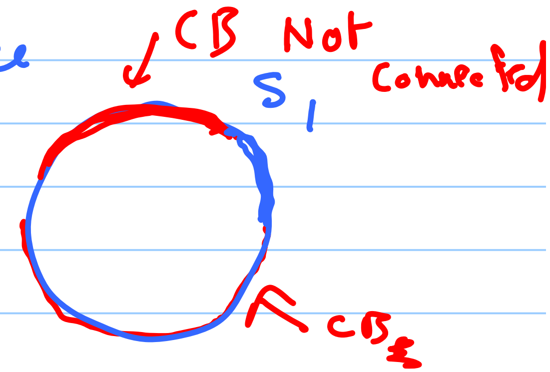
3) B is connected : is CB_i ?

A " "

yes for translation.
free



B



in Latombe : lot of such proof.
Ch 2, sec 9

4) B is regular : is CB_i ??

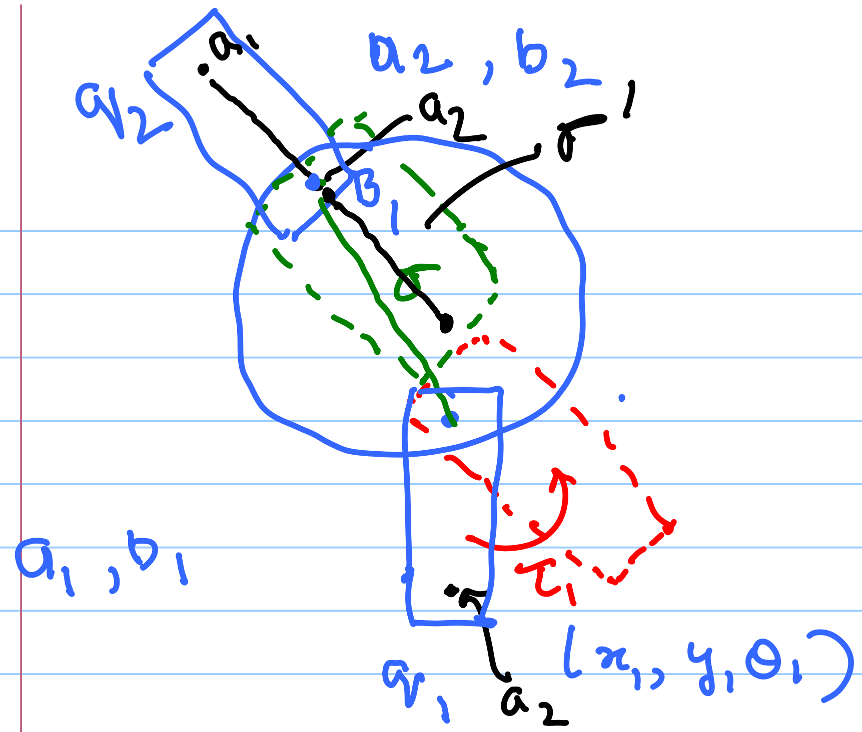
A is "

(x_2, y_2, θ_2)

$\tau(s) : \in CB_1 \ \forall s \in [0, 1]$

$q_1 \longrightarrow q_2$

$q_1 \in CB_1$ $\quad$ $q_2 \in CB_1$



$\tau_1 \cdot \underline{\tau_2} \cdot \tau_3$

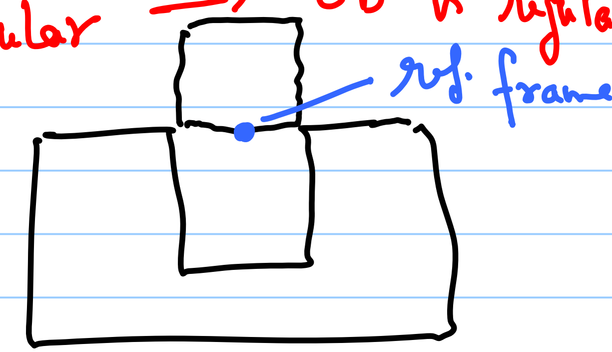
$=$

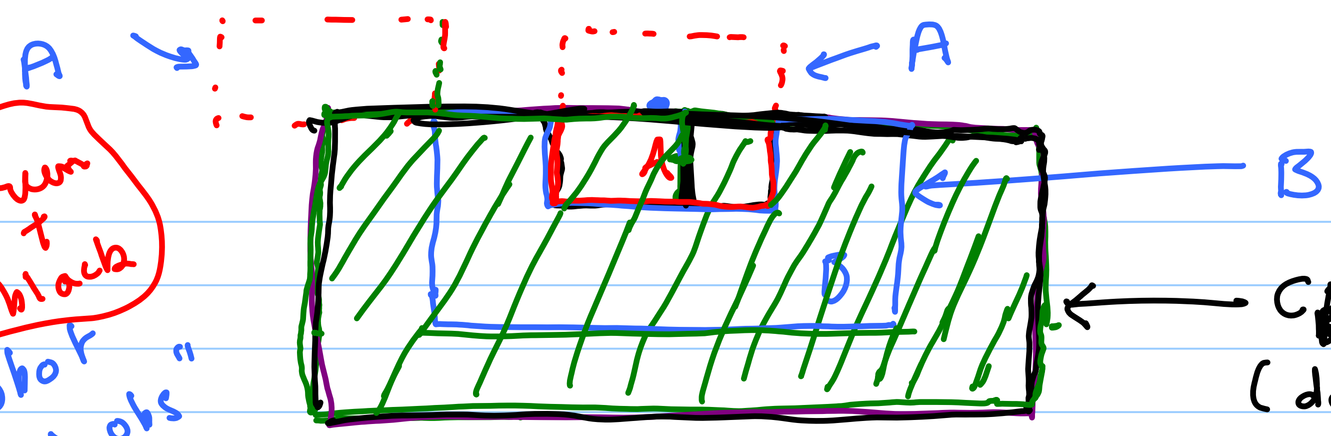
Regularity: A is regular, B is regular $\Longrightarrow$ CB is regular

reg. set

$\boxed{A = \overline{\text{Closure}(\text{int}(A))}}$

"no dangling edges"





pure trans

∂CB = no dang. edge

$CB_{contact}$ edge (dangling edge)

CB = green + black

"robot touching obs"

$$CB_{contact} = \{ q \in C : A(q) \cap B \neq \phi \wedge int(A(q)) \cap int(B) = \phi \}$$

robot overlapping obs

$$CB_{overlap} = \{ q : Int(A) \cap Int(B) \neq \phi \}$$

$$\boxed{CB_{contact} = CB \subseteq C_{overlap}}$$

$$\boxed{\partial CB \subseteq C_{contact}}$$

may have dangling edges

Recall by def. $\boxed{\partial S = cl(S) - Int(S)}$

$\boxed{CB = \text{Closure}(CB_{overlap})}$ ← A can translate freely at fixed orientation

proof see Latombe sec 9.3 ch. 2

semi-free path: "allow contact"

$\boxed{\tau : [0,1] \to Cl(C_{free})}$

general case
not est.
state clearly
under what
conditions
I can not to
check.

Th.: (Lounmond) if $\exists$ a semi-free path between two configs, then $\exists$ a path that is in C_{free} except ~~for~~ at isolated config.

($\in C_{valid}$)